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ENERGY EXCHANGE BETWEEN SEISMIC AND ULTRASONIC VIBRATIONS IN AN ELASTIC MEDIUM WITH A MICROSTRUCTURE†

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The flow rate of seismic energy under conditions of long-wave–short-wave resonance in granular geomaterials (soil or fissured mountain rock) is calculated using a continuous elastic non-linear model with microrotations. © 1997 Elsevier Science Ltd. All rights reserved.

We will use the equations for one-dimensional non-linear waves [1] in a grade-consistent elastic medium with microrotations [2]

$$u_{TT}-c_1^2u_{XX}-\delta u_{XXXX}-\nu u_Xu_{XX}+2\chi\varphi\varphi_X=0$$
$$\varphi_{TT}-c_2^2\varphi_{XX}+\omega_0^2\varphi-u_X(c_2^2\varphi_{XX}-\mu\varphi)=0 \tag{1}$$

to estimate the energy exchange in the case of long-wave–short-wave resonance (LSR). Here u is the displacement in the seismic wave, φ is the microrotation, c_1 and c_2 are, respectively, the nominal velocities of seismic and microrotational waves, and δ, ν, ω_0^2, χ, μ are the elastic and inertial coefficients. On the basis of a more detailed analysis of non-linear coefficients satisfying the grade-consistent theory [1], it can be shown that

$$\mu=\chi \tag{2}$$

We will restrict ourselves below to a linear analysis (with respect to energy!). The standard method yields the following expressions for the intensity of the seismic energy flux and the corresponding ultrasonic flux

$$W_s=c_1^2\omega_sk_s\rho u^2/2,\quad W_{us}=c_2^2\omega^*k^*\rho j\varphi^2/2 \tag{3}$$

Here ρ is the density, ρj is the specific moment of inertia of granules of the medium, k_s is the seismic wave number, $\omega_s^2=c_1^2k_s^2+\delta k_s^4$ is the dispersion relation for seismic waves and ω^* and k^* are the frequency and wave number of ultrasound excited during LSR [1].

The LSR condition has the form [1]

$$\frac{d\omega_{us}}{dk}(k^*)=c_1$$

where $\omega_{us}^2=c_2^2k^2+\omega_0^2$ is used as the dispersion relation for ultrasonic waves. In explicit form we have

$$c_2^2k^*[c_2^2(k^*)^2+\omega_0^2]^{-1/2}=c_1,\quad \omega^*=\omega_{us}(k^*)$$

We recall [1, 3] that in an LSR analysis the variables u and φ are represented in the form of series of powers of a small parameter

$$\varepsilon=\lambda_{us}^*/\lambda_s=k_s/k_{us}^*\ll 1$$

where $\lambda=2\pi/k$ is the wavelength. The "slow" variables ξ and τ are related to the initial "fast" variables X and T as follows:

$$\xi=\varepsilon(X-c_gT),\quad \tau=\varepsilon^2T,\quad c_g=d\omega_{us}/dk$$

Thus, taking into account the expansions in terms of a small parameter [1, 3]

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$$u = \varepsilon u_1 + O(\varepsilon^2), \quad \varphi = \varepsilon^q \varphi_1 + O(\varepsilon^{q+1}) \approx \varepsilon^q (Ae^{i\varphi} + \text{c.c.})$$

we can represent the energy fluxes (3) in the form

$$W_s = \varepsilon^2 \rho c_1^2 \omega_s k_s u_1^2 / 2, \quad W_{us} = \varepsilon^{2q} \rho j c_2^2 \omega^* k^* |A|^2 /2$$

We know [1, 3] that the LSR equations contain the seismic deformation $V = \partial u_1/\partial\xi \approx u_1 k_s$ instead of the displacement u_1, where the estimate containing the wave number can also be obtained from the asymptotic series [4]. Thus for $c_2 < c_1$, using the expressions for ultrasonic waves

$$k^* = \frac{c_1 \omega_0}{c_2 \sqrt{c_2^2 - c_1^2}}, \quad \omega^* = \frac{c_2 \omega_0}{\sqrt{c_2^2 - c_1^2}}$$

$$\omega^* k^* = \frac{c_1 \omega_0^2}{c_2^2 - c_1^2}, \quad \frac{\omega^*}{k^*} \equiv c_{us} = \frac{c_2^2}{c_1}$$

we have

$$W_{us} = \frac{1}{2} \rho j \omega_0^2 \frac{c_1 c_2^2}{c_2^2 - c_1^2} \frac{\varepsilon^3}{\chi_1} |A|^2$$

where [1]

$$\chi_1 = |\chi| c_1^{-2}, \quad \varepsilon^{2q} = \varepsilon^3 \chi_1^{-1} \tag{4}$$

and χ_1 is the reduced non-linearity coefficient. Note that the initial non-linearity coefficient (2) is assumed to be small, and the power exponent q is chosen to satisfy the transformation (4).

If $k_s^4 \ll k_s^2 c_1^2/\delta$, for seismic waves we have

$$W_s = \varepsilon^2 \rho c_1^2 \omega_s k_s V^2 k_s^{-2} / 2 = \varepsilon^2 \rho c_1^3 V^2 / 2$$

Thus, the required ratio of the intensities of the energy fluxes of high and low frequencies can be written in the form

$$\frac{W_{us}}{W_s} = \frac{j\omega_0^2}{c_1^2} \frac{c_2^2}{c_2^2 - c_1^2} \frac{\varepsilon}{\chi_1} \frac{|A|^2}{V^2} \tag{5}$$

The complex amplitude $A(\xi, \eta)$ and deformation $V(\xi, \eta)$ satisfy the LSR equations thus obtained (after correcting the misprints in [1, 3])

$$\partial_\tau V = \pm c_1 \partial_\xi |A|^2, \quad 2i\omega^* \partial_\tau A + (c_2^2 - c_1^2)\partial_{\xi\xi} A = [(k^*)^2 c_2^2 + \mu] VA$$

Estimates for the values of A and V are found by changing to the "canonical" LSR system

$$\partial_t L = \pm \partial_x |S|^2, \quad 2i\partial_t S + \partial_{xx} S = 2LS \tag{6}$$

for which we make a scalar replacement of variables

$$t = \alpha\tau, \quad x = \beta\xi, \quad A = \varkappa S, \quad V = \lambda L$$

Here (again correcting the misprints in [1, 3])

$$\alpha = \frac{\omega_0 c_*^2}{c_2 \sqrt{c_2^2 - c_1^2}}, \quad \beta = \frac{\omega_0 c_*}{c_2^2 - c_1^2}, \quad \lambda = 2, \quad \varkappa^2 = \frac{2c_* \sqrt{c_2^2 - c_1^2}}{c_1 c_2}$$

$$c_*^2 = (1 - \mu_1) c_1^2 + \mu_1 c_2^2, \quad \mu_1 = \mu \omega_0^{-2}$$

The coefficients (2) were taken to be small: $\mu = \chi \ll 1$, and so the simpler relation $c_* = c_1$ can be used.

Finally, we have

$$\frac{W_{us}}{W_s} = K \frac{|S|^2}{L^2}, \quad K = \frac{j}{2} \frac{\omega_0^2}{c_1^2} \frac{c_2}{\sqrt{c_2^2 - c_1^2}} \frac{\varepsilon}{\chi_1} \tag{7}$$

where K is the coefficient of energy exchange.

Quantitative expressions for K can be obtained using the following values (which are typical of porous mountain rock)

$$c_1 = 10^3 \text{ m/s}, \quad c_2 = 1.5 \times 10^3 \text{ m/s}, \quad \lambda_s = 10 \text{ m}, \quad \lambda_{us} \sim d = 10^{-3}$$

$$\omega_0 = 10^3 \text{ s}^{-1}, \quad j \approx d^2 = 10^{-6} \text{ m}^2, \quad \chi_1 = 10^{-7}, \quad \varepsilon = \lambda_{us}/\lambda_s = 10^{-4}$$

(d is the typical length of granules). This gives an estimated order of magnitude $K \sim 10^{-3}$, corresponding to the gradual accumulation of the non-linear effect (ultrasonic energy) at the typical large distances of the range of actual seismic waves. As we know [5], ultrasound effectively influences the microstate of a two-phase mixture (such as oil and water) in the pore space of granular media, transforming the mixture to a micro-emulsive state, thereby improving the oil recovery of deposits.

We will now obtain an analytic estimate for the coefficient $|S|^2/L^2$ which appears in expression (7). By the replacement of variables $S = I^{1/2}e^{i\varphi}$, $\partial_x\varphi = w$ [1], Eqs (6) take the form encountered in fluid dynamics

$$\partial_t I + \partial_x (Iw) = 0, \quad \partial_t L + \partial_x I = 0$$

$$\partial_t w + \partial_x \left(\frac{w^2}{2} + L \right) + \frac{1}{4} \partial_x \left[\frac{(\partial_x I)^2}{2I^2} - \frac{\partial_{xx} I}{I} \right] = 0 \tag{8}$$

In the limit of weak dispersion $\partial_x I/I \ll 1$, derivatives of the second and third orders can be neglected. Then system (8) can be put in Riemann form [1]

$$\partial_t r_i + V_i(\mathbf{r}) \partial_x r_i = 0 \quad (i = 1,2,3, \ \mathbf{r} = (r_1, r_2, r_3)) \tag{9}$$

if we introduce the new variables

$$r_i = L + w^2 (3\Lambda_i - 2)\Lambda_i / 2 \tag{10}$$

where Λ_i are the roots of the equation

$$\Lambda(\Lambda - 1)^2 = Iw^{-3} \tag{11}$$

(Fig. 1), and $V_i = w\Lambda_i$. If $0 < Iw^{-3} < 4/27$, all the roots of Eq. (11) are real and system (9) is hyperbolic.

We will use the combination $p = \Lambda_1(Iw^{-3})$ as a parameter. For given p the roots Λ_2, Λ_3 satisfy a quadratic equation and thus

$$\Lambda_{2,3} = 1 - p/2 \mp \sqrt{p(4 - 3p)}/2$$

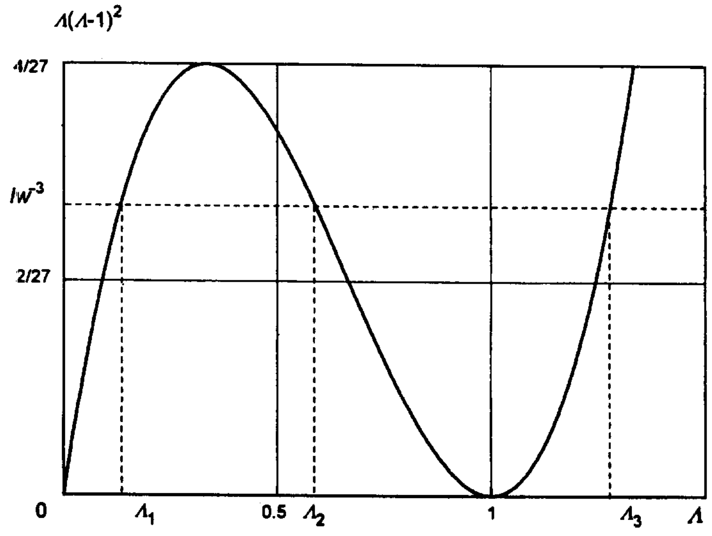

Fig. 1.

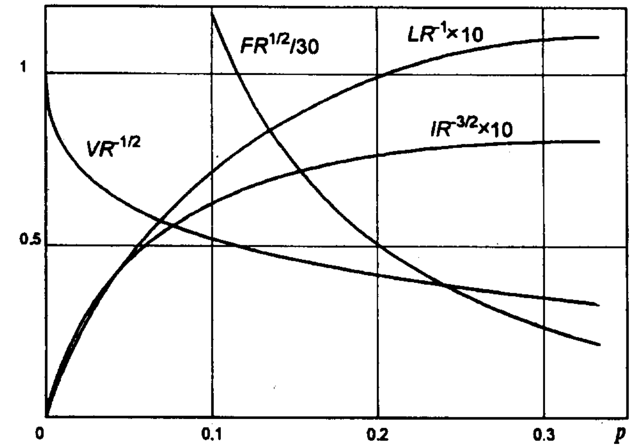

Fig. 2.

Substituting these expressions into (10), we obtain

$$r_i = L + w^2 h_i(p), \quad i = 1,2,3 \tag{12}$$

$$h_1(p) = \frac{(3p-2)p}{2}, \quad h_{2,3}(p) = \frac{1}{4}[2 + 2p - 3p^2 \mp (4-3p)\sqrt{p(4-3p)}]$$

Let us consider the particular solutions of (9) for $r_1 = 0$, $r_3 = R =$ const. These are known as simple Riemann waves, in which all the quantities are functions of one variable, p say, found from other equation $\partial_t p + V(p)\partial_x p = 0$, where $V(p) = V_2(0, r_2(p), R) = w(p)\Lambda_2(p)$.

The unknown values in (12), expressed in terms of p (under conditions $r_1 = 0$, $r_3 = R$), are

$$w^2 = RD^{-1}, \quad L = -Rh_1 D^{-1}, \quad I = R^{3/2} p(p-1)^2 D^{-3/2}$$

$$r_2 = R(h_2 - h_1)D^{-1}$$

$$V = \tfrac{1}{2} R^{1/2}[2 - p - \sqrt{p(4-3p)}]^{1/2} D^{-1/2}$$

$$F \equiv \frac{|S|^2}{L^2} = \frac{I}{L^2} = 4R^{-1/2}\frac{(p-1)^2}{p(3p-2)^2} D^{1/2}$$

$$D = h_3 - h_1 = \tfrac{1}{4}[2 + 6p - 9p^2 + (4-3p)\sqrt{p(4-3p)}]$$

Figure 2 shows graphs of the functions $L(p)$, $I(p)$, $V(p)$, $F(p)$.

Note that the unlimited increase in F as $p \to 0$ (or $L \to 0$) has no physical interpretation, because it invalidates the series of [1] required for LSR.

A reduction in the values of $V(p)$ (and naturally also $V(L)$) stabilizes the leading edge of the wave, which enters the medium with increasing amplitude, whereas the tail of the wave breaks up. In that case the dispersion terms must be taken into account in the LSR system.

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